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Sem-II

Mathematical & Statistical Techniques-II

Anil Khadse Sheth NKTTC College, Thane

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Syllabus Sem-II

Mathematics

Module-I : Derivatives & its Applications Marks:20

Module-II : -Interest
 -Annuity Marks:20

Statistics

Module-III : -Correlation
 - Regression Marks:20

Module-IV : -Index Number
 -Time series Marks:20

Module-V : Probability Distribution Marks:20

Module-III

Correlation & Regression

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Correlation

Correlation means Finding the relationship between two quantitative variables without being able to infer causal relationships

Correlation is a statistical technique used to determine the degree to which two variables are related

e.g Demand & Supply,
Income & Expenditure
Age and Height of Children
Study Hours & Score

- Are two variables related?
 - Does one increase as the other increases?
 - e. g. skills and income
 - Does one decrease as the other increases?
 - e. g. health problems and nutrition
- How can we get a numerical measure of the degree of relationship?

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Types of Correlation

Positive Correlation:

If two series move in same direction that is one increases other also increases or both decreases, there is positive correlation between the variables.

Example:

Income & Expenditure

Study hrs & score

Age of Husband & wife

Negative Correlation:

If two series move in opposite direction that is one increases other decreases or vice a versa ,there is negative correlation between the variables.

Example:

Price & Demand

Methods to determine Correlation

- Scatter Diagram
- Karl Pearson's Correlation coefficient
- Spearman's Rank Correlation Coefficient

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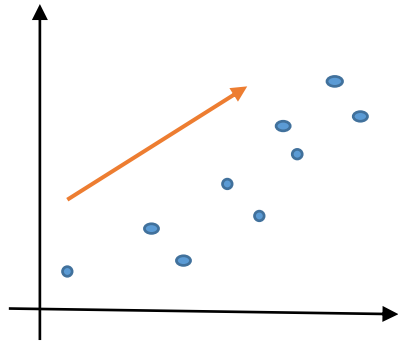
Scatter Diagram

Scatter Diagram gives us the idea about existence of the relation between variables.

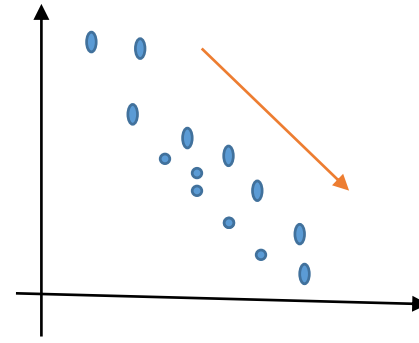
In scatter diagram one of the variable is consider on X-axis and other on Y-axis.

The points are plotted on graph and from the direction of the movement of the points we can conclude on the relationship

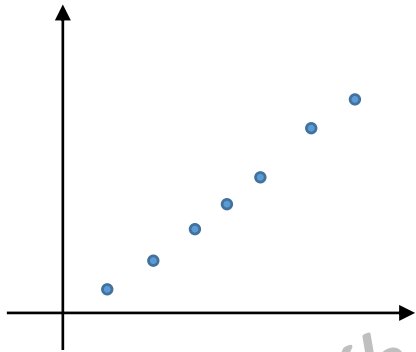
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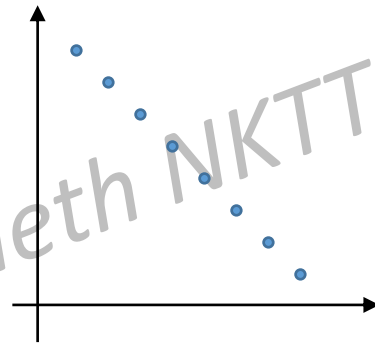
Positive Correlation



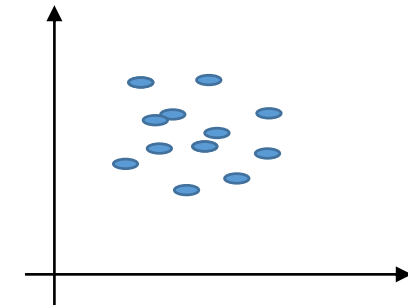
Negative Correlation



Perfect Positive

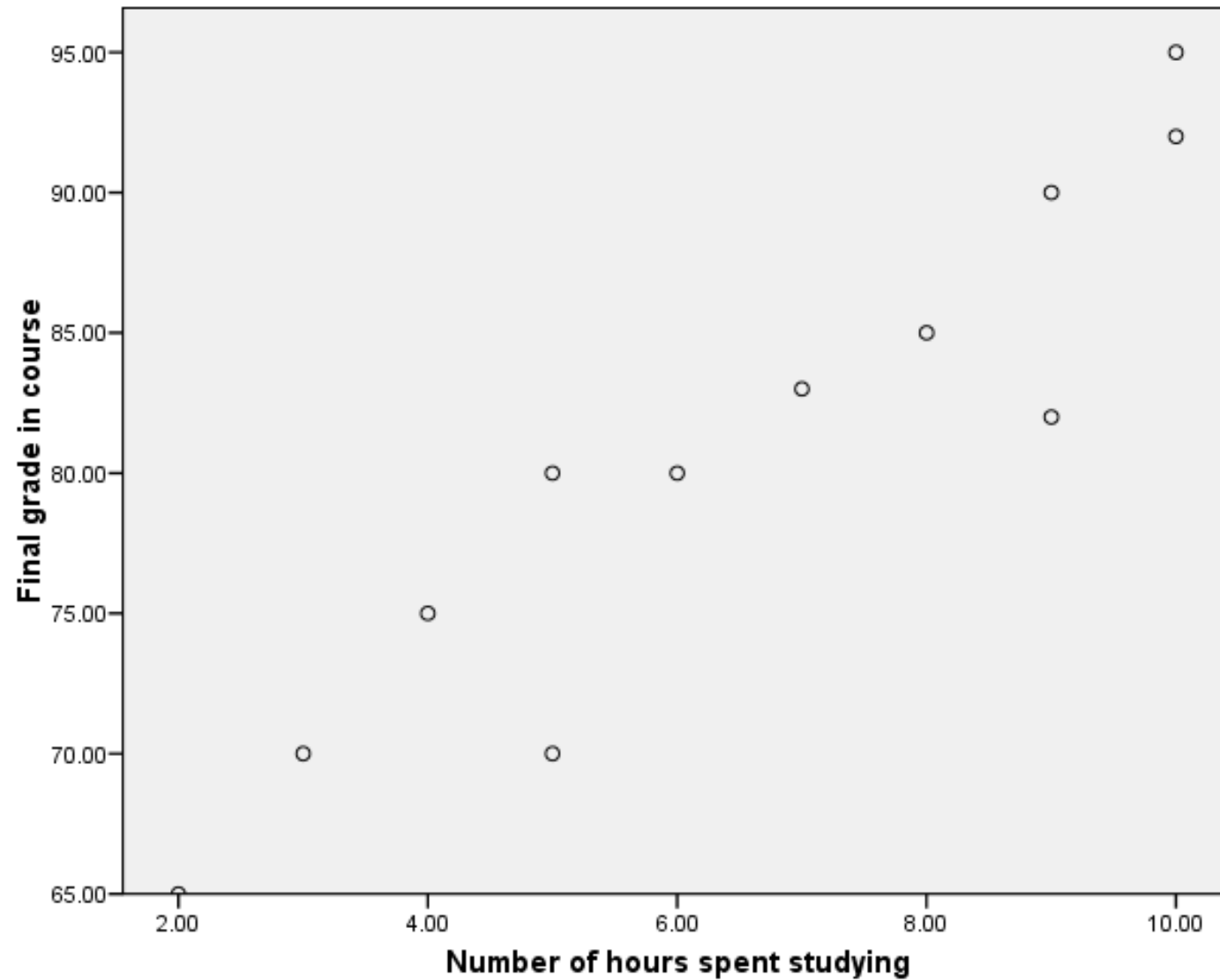


Perfect Negative



No Correlation

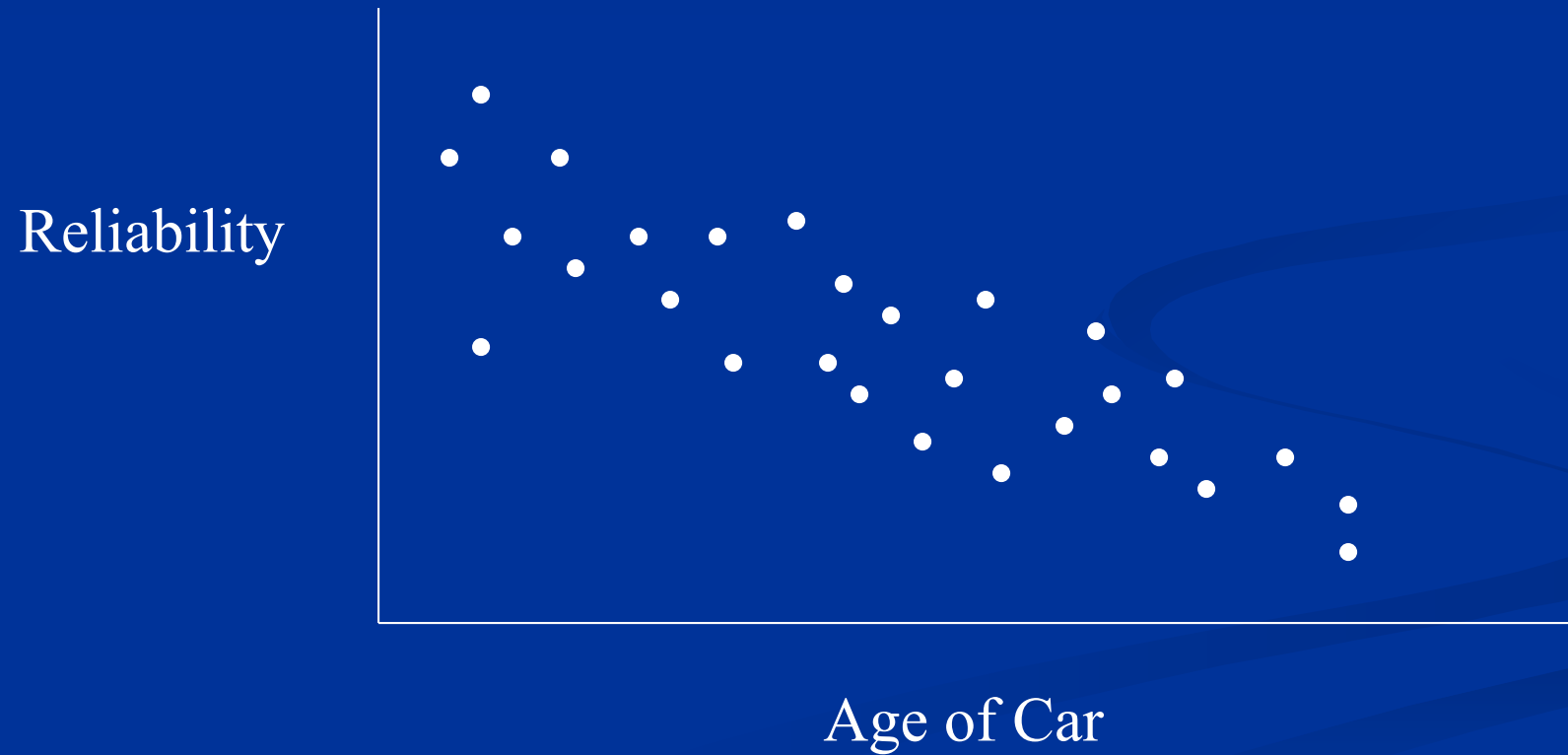
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Positive Correlation

Negative relationship



Karl Pearson's Coefficient of correlation (r)/Product Moment

If $(x_1, y_1), (x_2, y_2) \dots (x_n, y_n)$ are n pairs of observations of two variable X and Y , Karl Pearson Coeff. of correlation is denoted by r and defined as

$$r = \frac{\sum(x - \bar{x})(y - \bar{y})}{n \cdot \sigma_x \cdot \sigma_y} \quad \text{--- (i)}$$

Where, \bar{x} = Mean of x values

\bar{y} = Mean of y values

σ_x = s.d of x values

σ_y = s,d of y values

$$r = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2} \cdot \sqrt{\sum(y-\bar{y})^2}} \text{----- (ii)}$$

$$r = \frac{Cov(x,y)}{n \cdot \sigma_x \cdot \sigma_y} \text{----- (iii)}$$

After substituting the values of mean and s.d

$$r = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sqrt{\sum x^2 - \frac{(\sum x)^2}{n}} \cdot \sqrt{\sum y^2 - \frac{(\sum y)^2}{n}}} \text{----- (iv)}$$

Interpretation of r :

- The value of r ranges between (-1) and (+1)
- The value of r denotes the strength of the association as illustrated by the following diagram.



1. If $0 < r < 1$, then positive correlation
2. If $-1 < r < 0$, then Negative correlation
3. If $r = +1$, then Perfect positive correlation
4. If $r = -1$, then Perfect negative correlation
5. If $r = 0$, then No correlation

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Example # 1

Calculate Karl Pearson's coefficient of correlation for the following data

X	12	10	8	13	7
Y	15	20	25	18	22

X	y	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})^2$	$(y - \bar{y})^2$	$(x - \bar{x})(y - \bar{y})$
12	15	2	-5	4	25	-10
10	20	0	0	0	0	0
8	25	-2	5	4	25	-10
13	18	3	-2	9	4	-6
7	22	-3	2	9	4	-6
50	100			26	58	-32

$$r = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2} \cdot \sqrt{\sum(y-\bar{y})^2}}$$

$$r = \frac{-32}{\sqrt{26} \cdot \sqrt{58}}$$

$$r = \frac{-32}{5.099 \times 7.6157}$$

$$r = \frac{-32}{38.8324}$$

$$r = -0.8240$$

There is -ve correlation

Example # 2

Find Karl Pearson's coefficient of correlation for the following data

serial No	Age (years)	Weight (Kg)
1	7	12
2	6	8
3	8	12
4	5	10
5	6	11
6	9	13

Sr.No	x	y	xy	X²	Y²
1	7	12	84	49	144
2	6	8	48	36	64
3	8	12	96	64	144
4	5	10	50	25	100
5	6	11	66	36	121
6	9	13	117	81	169
Total	$\Sigma x =$ 41	$\Sigma y =$ 66	$\Sigma xy =$ 461	$\Sigma x^2 =$ 291	$\Sigma y^2 =$ 742

$$r = \frac{\sum xy - \sum x \sum y / n}{\sqrt{\sum x^2 - \frac{(\sum x)^2}{n}} \cdot \sqrt{\sum y^2 - \frac{(\sum y)^2}{n}}}$$

$$r = \frac{461 - \frac{41 \times 66}{6}}{\sqrt{\left[291 - \frac{(41)^2}{6}\right]} \cdot \sqrt{\left[742 - \frac{(66)^2}{6}\right]}}$$

$$r = \frac{461 - 451}{\sqrt{291 - 280.1666} \cdot \sqrt{742 - 726}}$$

$$r = \frac{10}{\sqrt{10.8334} \cdot \sqrt{16}}$$

$$r = \frac{10}{3.2914 \times 4}$$

$$r = \frac{10}{13.1656}$$

$$r = 0.7595$$

Example #3 : Calculate Coefficient of correlation from the following information

$$n = 12, \Sigma x = 35, \Sigma y = 60, \Sigma x^2 = 148, \Sigma y^2 = 450, \Sigma xy = 105$$

$$r = \frac{\Sigma xy - \Sigma x \Sigma y / n}{\sqrt{\Sigma x^2 - \frac{(\Sigma x)^2}{n}} \cdot \sqrt{\Sigma y^2 - \frac{(\Sigma y)^2}{n}}}$$

$$r = \frac{105 - 35 \times 60 / 12}{\sqrt{148 - \frac{(35)^2}{12}} \cdot \sqrt{450 - \frac{(60)^2}{12}}}$$

$$r = \frac{105 - 175}{\sqrt{148 - \frac{1225}{12}} \cdot \sqrt{450 - \frac{3600}{12}}}$$

$$r = \frac{-70}{\sqrt{148 - 102.0833} \cdot \sqrt{450 - 300}}$$

$$r = \frac{-70}{\sqrt{45.9187} \cdot \sqrt{150}}$$

$$r = \frac{-70}{6.7763 \cdot 12.2474}$$

$$r = \frac{-70}{82.9923}$$

$$r = -0.84$$

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Example #4: Calculate Coefficient of correlation from the following results
 $n = 8, \sum(x - \bar{x})^2 = 84, \sum(y - \bar{y})^2 = 158, \sum(x - \bar{x})(y - \bar{y}) = 111$

$$r = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sqrt{\sum(x - \bar{x})^2} \cdot \sqrt{\sum(y - \bar{y})^2}}$$

$$r = \frac{111}{\sqrt{84} \cdot \sqrt{158}}$$

$$r = \frac{111}{9.1651 \cdot 12.5698}$$

$$r = \frac{111}{115.2035}$$

$$r = 0.9635$$

Example #5: Calculate Coefficient of correlation from the following results
 $n = 8$, *s.d of x* = 3.86, *s, d of y* = 6.57, $\sum(x - \bar{x})(y - \bar{y}) = 192$

$$r = \frac{\sum(x - \bar{x})(y - \bar{y})}{n \cdot \sigma_x \cdot \sigma_y}$$

$$r = \frac{192}{8 \times 3.86 \times 6.57}$$

$$r = \frac{192}{202.8816}$$

$$r = 0.9463 = 0.95$$

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Exercise # 1

Calculate Karl Pearson's coefficient of correlation for the following data

X	14	8	10	11	9	13	5
Y	14	9	11	13	11	12	4

Ans: 0.9231

Excercise#2 : Calculate Coefficient of correlation from the following information

$$n = 25, \Sigma x = 75, \Sigma y = 100, \Sigma x^2 = 250, \Sigma y^2 = 500, \Sigma xy = 325$$

Example #3: Calculate Coefficient of correlation from the following results

$$n = 8, \Sigma (x - \bar{x})^2 = 100, \Sigma (y - \bar{y})^2 = 195, \Sigma (x - \bar{x})(y - \bar{y}) = 85$$

Spearman's Rank Correlation Coefficient (R)

It is applicable for quantitative as well as qualitative data

The given values of the variable are assigned the rank in order, and these rank provides data to calculate correlation. It is denoted by R and defined as

$$R = 1 - \frac{6 \sum d^2}{n(n^2 - 1)}$$

Where, $d = R1 - R2$: *Difference between ranks*

$R1 =$ *Rank for first variable or x*

$R2 =$ *Rank for Second variable or y*

$n =$ *No. of pairs of observation*

Example #1: Calculate Rank correlation coefficient from the following

Rank by Judge -A	5	2	1	4	3
Rank by Judge-B	4	2	3	5	1

R1	R2	d=R1-R2	d ²
5	4	1	1
2	2	0	0
1	3	-2	4
4	5	-1	1
3	1	2	4
			$\Sigma d^2 = 10$

$$n = 5$$

$$R = 1 - \frac{6 \sum d^2}{n(n^2 - 1)}$$

$$R = 1 - \frac{6 \times 10}{5(5^2 - 1)}$$

$$R = 1 - \frac{60}{5(25 - 1)}$$

$$R = 1 - \frac{60}{5(24)}$$

$$R = 1 - \frac{60}{120}$$

$$R = 1 - 0.5 = 0.5$$

Example #2: Find Rank correlation coefficient from the following data

Demand	15	22	20	30	25
Supply	18	25	26	28	20

Sol: Let R1 denotes rank for demand and R2 denotes rank for supply

x	y	R1	R2	d	d ²
15	18	5	5	0	0
22	25	3	3	0	0
20	26	4	2	2	4
30	28	1	1	0	0
25	20	2	4	-2	4
					$\Sigma d^2 = 8$

$$n = 5$$

$$R = 1 - \frac{6 \sum d^2}{n(n^2 - 1)}$$

$$R = 1 - \frac{6 \times 8}{5(5^2 - 1)}$$

$$R = 1 - \frac{48}{5(25 - 1)}$$

$$R = 1 - \frac{48}{5(24)}$$

$$R = 1 - \frac{48}{120}$$

$$R = 1 - 0.4 = 0.6$$

Example# 3

The coefficient of rank correlation between marks in two subjects obtained by a group of students is 0.8. If the sum of squares of the differences in ranks is 33. Find the number of students in the group.

Given: Rank correlation $R = 0.8$

$$\sum d^2 = 33, n = ?$$

$$R = 1 - \frac{6 \sum d^2}{n(n^2 - 1)}$$

$$0.8 = 1 - \frac{6 \times 33}{n(n^2 - 1)}$$

$$0.8 - 1 = -\frac{6 \times 33}{n(n^2 - 1)}$$

$$-0.2 = -\frac{198}{n(n^2 - 1)}$$

$$n(n^2 - 1) = 198 / 0.2$$

$$n(n^2 - 1) = 198 / 0.2 = 990$$

$$n(n^2 - 1) = 10(10^2 - 1)$$

$$n = 10$$

Rank correlation for repeated values :

If the values are repeated then the common rank is assigned by considering average of the ranks to all repeated values.

If the values are repeated then the correction factor is added to $\sum d^2$ to find rank correlation

$$C.F = \frac{\sum m(m^2-1)}{12}$$

$$C.F = \frac{1}{12} [(m_1(m_1^2-1)+m_2(m_2^2-1)+\dots)]$$

$$R = 1 - \frac{6(\sum d^2 + C.F)}{n(n^2-1)}$$

Example #2: Find Rank correlation coefficient from the following data

X	25	20	20	18	32	35
y	58	55	48	62	55	40

Sol: Let R1 denotes rank for demand and R2 denotes rank for supply

x	y	R1	R2	d	d ²
25	58	3	2	1	1
20	55	4.5	3.5	1	1
20	48	4.5	5	-0.5	0.25
18	62	6	1	5	25
32	55	2	3.5	-1.5	2.25
35	40	1	6	-5	25

20 repeated twice
 Rank = $4+5/2=4.5$
 $m_1 = 2$

55 repeated twice
 Rank = $3+4/2=3.5$
 $m_2 = 2$

$$\Sigma d^2 = 54.58$$

$$C.F = \frac{1}{12} [(m_1(m_1^2-1)+m_2(m_2^2-1))]$$

$$C.F = \frac{1}{12} [(2(2^2-1)+2(2^2-1))]$$

$$C.F = \frac{1}{12} [(2(4-1)+2(4-1))]$$

$$C.F = \frac{1}{12} [6+6]=12/12=1$$

$$R = 1 - \frac{6(\sum d^2 + C.F)}{n(n^2-1)}$$

$$R = 1 - \frac{6(54.5+1)}{6(6^2-1)}$$

$$R = 1 - \frac{333}{6(36-1)}$$

$$R = 1 - \frac{333}{6(35)}$$

$$R = 1 - \frac{333}{(210)}$$

$$R = 1 - 1.5857$$

$$R = -0.5857$$

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Regression

- Regression method is use to predict / Estimate one variable (dependent) when the value of independent variable is known
 - Tells you how values in y change as a function of changes in values of x
1. Businessman wants to know effect of increase in advertising on sales.
 2. To find the effect of change in demand pattern of some commodities on price

Types of Regression

Regression Equation of y on x

Regression Equation of x on y

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1. Regression equation of y on x

It is used to estimate the value of dependant variable y when the value of independent variable x is given

Regression equation of y on x is given by

$$y = a + bx$$

$$y - \bar{y} = byx(x - \bar{x})$$

$$y = byx(x - \bar{x}) + \bar{y}$$

\bar{x} = mean of X, \bar{y} = mean of Y

byx = regression coeff of y on x

$$byx = r \cdot \frac{\sigma_y}{\sigma_x}$$

$$byx = \frac{\sum xy - \frac{\sum x \cdot \sum y}{n}}{\sum x^2 - \frac{(\sum x)^2}{n}}$$

2. Regression equation of x on y

It is used to estimate the value of dependant variable x when the value of independent variable y is given

Regression equation of x on y is given by

$$x = a + by$$

$$x - \bar{x} = bxy(y - \bar{y})$$

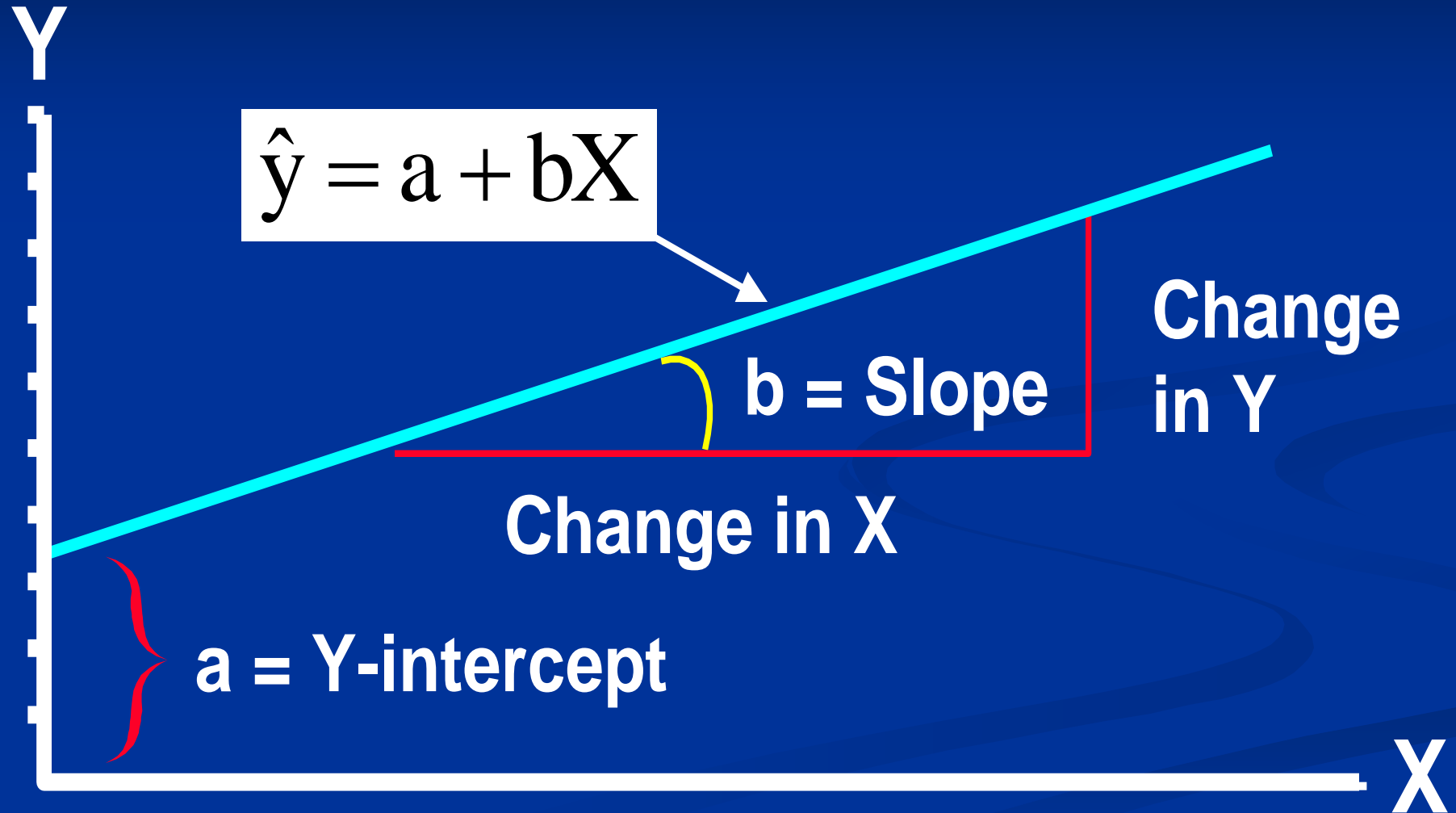
$$x = bxy(y - \bar{y}) + \bar{x}$$

bxy = regression coeff of x on y

$$bxy = r \cdot \frac{\sigma_x}{\sigma_y}$$

$$bxy = \frac{\sum xy - \frac{\sum x \cdot \sum y}{n}}{\sum y^2 - \frac{(\sum y)^2}{n}}$$

Linear Equations



Example #1 Find two regression equations from the following data

X	8	7	10	9	5	6
y	11	8	12	13	8	10

x	y	x^2	y^2	xy
8	11	64	121	88
7	8	49	64	56
10	12	100	144	120
9	13	81	169	117
5	8	25	64	40
6	10	36	100	60
$\Sigma x =$ 45	$\Sigma y =$ 62	$\Sigma x^2 =$ 355	$\Sigma y^2 =$ 662	$\Sigma xy =$ 481

n=6

$$\bar{x} = \frac{\sum x}{n} = \frac{45}{6} = 7.5$$

$$\bar{y} = \frac{\sum y}{n} = \frac{62}{6} = 10.33$$

$$b_{yx} = \frac{\sum xy - \frac{\sum x \cdot \sum y}{n}}{\sum x^2 - \frac{(\sum x)^2}{n}}$$

$$b_{yx} = \frac{481 - 45 \times 62/6}{355 - \frac{(45)^2}{6}}$$

$$b_{yx} = \frac{481 - 465}{355 - 337.5}$$

$$b_{yx} = \frac{16}{17.5} = 0.91$$

$$b_{xy} = \frac{\sum xy - \frac{\sum x \cdot \sum y}{n}}{\sum y^2 - \frac{(\sum y)^2}{n}}$$

$$b_{xy} = \frac{16}{662 - \frac{(62)^2}{6}}$$

$$b_{xy} = \frac{16}{662 - 640.666}$$

$$b_{xy} = \frac{16}{21.34} = 0.75$$

Regression equation of y on x is given by

$$y = b_{yx}(x - \bar{x}) + \bar{y}$$

$$y = 0.91(x - 7.5) + 10.33$$

$$y = 0.91x - 0.91 \times 7.5 + 10.33$$

$$y = 0.91x - 6.825 + 10.33$$

$$y = 0.91x + 3.505$$

Regression equation of x on y is given by

$$x = b_{xy}(y - \bar{y}) + \bar{x}$$

$$x = 0.75(y - 10.33) + 7.5$$

$$x = 0.75y - 0.75 \times 10.33 + 7.5$$

$$x = 0.75y - 7.7475 + 7.5$$

$$x = 0.75y - 0.2475$$

Example # 2

For bivariate distribution, Mean of $x = 65$, Mean of $y = 53$
s.d of $x = 4.7$ s.d of $y = 5.2$, Correlation Coeff = 0.78

Find two regression equations and estimate

- i) The most probable value of y when $x = 63$
- ii) The most probable value of x when $y = 50$

Given: $\bar{x} = 65$, $\bar{y} = 53$, $\sigma_x = 4.7$, $\sigma_y = 5.2$, $r = 0.78$

$$b_{yx} = r \cdot \frac{\sigma_y}{\sigma_x} = 0.78 \cdot \frac{5.2}{4.7} = 0.86$$
$$b_{xy} = r \cdot \frac{\sigma_x}{\sigma_y} = 0.78 \cdot \frac{4.7}{5.2} = 0.705 = 0.71$$

Regression equation of y on x is given by

$$y = byx(x - \bar{x}) + \bar{y}$$

$$y = 0.86(x - 65) + 53$$

$$y = 0.86x - 0.86 \times 65 + 53$$

$$y = 0.86x - 55.9 + 53$$

$$y = 0.86x - 2.9$$

When x=63

$$y = 0.86 \times 63 - 2.9$$

$$y = 51.28$$

$$y = 54.18 - 2.9$$

Regression equation of x on y is given by

$$x = b_{xy}(y - \bar{y}) + \bar{x}$$

$$x = 0.71(y - 53) + 65$$

$$= 0.71y - 0.71 \times 53 + 65$$

$$= 0.71y - 37.63 + 65$$

$$x = 0.71y + 27.37$$

When $y=50$

$$x = 0.71 \times 50 + 27.37$$

$$x = 62.87$$

$$x = 35.5 + 27.37$$

Example # 3

For bivariate distribution, Mean of $x = 43$, Mean of $y = 37$

Regression coeff. of y on $x = 0.59$

Regression Coeff of x on $y = 0.72$

Find two regression equations and estimate

i) Likely value of y when $x = 40$

ii) Likely value of x when $y = 35$

Given: $\bar{x} = 43$, $\bar{y} = 37$, $b_{yx} = 0.59$, $b_{xy} = 0.72$,

Regression equation of y on x is given by

$$y = b_{yx}(x - \bar{x}) + \bar{y}$$

$$y = 0.59(x - 43) + 37$$

$$y = 0.59x - 0.59 \times 43 + 37$$

$$y = 0.59x - 25.37 + 37$$

$$y = 0.59x + 11.63$$

When $x=40$

$$y = 0.59 \times 40 + 11.63$$

$$y = 23.6 + 11.63 = 35.23$$

Regression equation of x on y is given by

$$x = b_{xy}(y - \bar{y}) + \bar{x}$$

$$x = 0.72(y - 37) + 43$$

$$x = 0.72y - 0.72 \times 37 + 43$$

$$x = 0.72y - 26.64 + 43$$

$$x = 0.72y + 16.36$$

When $y = 35$

$$x = 0.72 \times 35 + 16.36$$

$$x = 41.56$$

Example # 4

Given the following data, find two regression equations. Also estimate y if $x=60$ and x if $y = 37$

	x	Y
Mean	65	39
s.d	4.3	1.2

Correlation Coefficient = 0.75

Given: $\bar{x} = 65$, $\bar{y} = 39$, $\sigma_x = 4.3$, $\sigma_y = 1.2$, $r = 0.75$

$$b_{yx} = r \cdot \frac{\sigma_y}{\sigma_x} \quad b_{yx} = 0.75 \times \frac{1.2}{4.3} \quad b_{yx} = 0.21$$

$$b_{xy} = r \cdot \frac{\sigma_x}{\sigma_y} \quad b_{xy} = 0.75 \times \frac{4.3}{1.2} = 2.69$$

Regression equation of y on x is given by

$$y = 0.21(x - 65) + 39$$

$$y = 0.21x - 13.65 + 39$$

$$y = 0.21x + 25.35$$

$$y = 0.21 \times 60 + 25.35$$

Put x = 60

$$y = 37.95$$

Regression equation of x on y is given by

$$x = b_{xy}(y - \bar{y}) + \bar{x}$$

$$x = 2.69(y - 39) + 65$$

$$x = 2.69y - 104.91 + 65$$

$$x = 2.69y - 39.91$$

$$x = 2.69 \times 37 - 39.91$$

Put $y = 37$

$$x = 99.53 - 39.91$$

$$x = 59.62$$

Properties of regression lines

- The two regression lines coincide if there is perfect +ve or perfect -ve correlation between the variables.
- The two regression lines are perpendicular to each other if there is no correlation between the variables.
- *The point (\bar{x}, \bar{y}) satisfies both the regression equations*
- Relation between correlation and regression coefficients is

$$r = \pm \sqrt{byx \cdot bxy}$$

Sign of corr coeff. r depends on the sign of regression coefficients.

- i) r is positive if both the regression coefficients are positive
- ii) r is negative if both the regression coefficients are negative

From the given regression equations, regression coefficients can be obtained as

From reg. equation of y on x

$$b_{yx} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } y}$$

From reg. equation of x on y

$$b_{xy} = -\frac{\text{Coefficient of } y}{\text{Coefficient of } x}$$

Example # 5

The regression equation of y on x is $x + 3y - 88 = 0$

and that of regression equation of x on y is $2x + y - 71 = 0$.

Find i) Mean values of x and y

ii) Coefficient of correlation

SOL: To find mean values, solve two regression equations

$$x + 3y - 88 = 0 \text{ ----- (i) } \times 2$$

$$2x + y - 71 = 0 \text{ ----- (ii)}$$

$$2x + 6y - 176 = 0 \text{ ----- (i)}$$

$$2x + y - 71 = 0 \text{ ----- (ii)}$$

$$\begin{array}{r} - \quad - \quad + \\ \hline \end{array}$$

$$5y = 105$$

$$y = 21$$

Put $y = 21$ in equation (i)

$$x + 3y - 88 = 0$$

$$x + 3 \times 21 - 88 = 0$$

$$x + 63 - 88 = 0$$

$$x = 25$$

$$\bar{x} = 25, \bar{y} = 21$$

ii) To find r

The regression equation of y on x is $x + 3y - 88 = 0$

$$b_{yx} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } y}$$

$$b_{yx} = -\frac{1}{3}$$

Regression equation of x on y is $2x + y - 71 = 0$.

$$b_{xy} = -\frac{\text{Coefficient of } y}{\text{Coefficient of } x} = -\frac{1}{2}$$

$$r = \pm \sqrt{b_{yx} \cdot b_{xy}}$$

$$r = \pm \sqrt{-\frac{1}{3} \times -\frac{1}{2}} = -\sqrt{\frac{1}{6}}$$

$$r = -\sqrt{0.1666}$$

$$r = -0.4082$$

Example # 6

Given two regression equations $2x + 3y = 5$ and $x + y = 2$.

Find i) Mean values of x and y

ii) Coefficient of correlation

SOL: To find mean values, solve two regression equations

$$2x + 3y = 5 \quad \text{-----} \quad \text{---(i)}$$

$$x + y = 2 \quad \text{-----} \quad \text{---(ii) } \times 2$$

$$2x + 3y = 5$$

$$2x + 2y = 4 \quad \text{subtract ii from i}$$

$$\begin{array}{r} - \quad - \\ \hline y = 1 \end{array}$$

Put $y = 1$ in equation ii $x + 1 = 2 \quad \Rightarrow x = 1$

$$\bar{x} = 1 \text{ \& } \bar{y} = 1$$

ii) To find r

Let regression equation of y on x is $2x + 3y = 5$

$$b_{yx} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } y} \quad b_{yx} = -\frac{2}{3}$$

Regression equation of x on y is $x + y = 1$.

$$b_{xy} = -\frac{\text{Coefficient of } y}{\text{Coefficient of } x} \quad b_{xy} = -\frac{1}{1} = -1$$

$$r = \pm \sqrt{b_{yx} \cdot b_{xy}}$$

$$r = \pm \sqrt{-2/3 \times -1}$$

$$r = -\sqrt{0.6666}$$

$$r = -0.8165$$

Exercise#1 Find two regression equations from the following data and Estimate y when x=16 and x when y 18

X	3	4	6	10	12	13
y	12	11	15	16	19	17

Example # 2

For bivariate distribution, Mean of x = 25, Mean of y = 152

s.d of x = 1.8 s.d of y = 5.7, Correlation Coeff = 0.82

Find two regression equations and estimate y if x = 23 and x if y = 145

Example # 3

Given the following data, find two regression equations. Also estimate age of wife if age of husband is 25.

	Husband	Wife
Average age	27 years	23 years
s.d	3 years	2 years

Correlation Coefficient = 0.75

Example # 6

Given two regression equations $3x - y - 25 = 0$ and $2x - 3y + 30 = 0$.

Find i) Mean values of x and y

ii) Coefficient of correlation

iii) s.d of x if s.d of y is 2

$$b_{xy} = r \cdot \frac{\sigma_x}{\sigma_y}$$

$$1/3 = 0.4714 \cdot \frac{\sigma_x}{2}$$

$$2/3 = 0.4714 \cdot \sigma_x$$

$$1.41 = \sigma_x$$